Domácí cvičení č.5

15. Určete $\mathbf{T}$ matici přechodu od báze $f_1, f_2, \dots$ k bázi $g_1, g_2, \dots$ prostoru $\mathcal{L}$ a $\mathbf{T^{-1}}$ matici přechodu od báze $g_1, g_2, \dots$ k bázi $f_1, f_2, \dots$.

    (a) $\mathcal{L} = \mathbb{R}_2$,
    $f_1 = [1,2]^T$, $f_2 = [2,1]^T$,
    $g_1 = [3,5]^T$, $g_2 = [5,3]^T$,

    $$[\mathbf{T} = \frac{1}{3}\begin{bmatrix} 7 & 1 \\ 1 & 7 \end{bmatrix}, \mathbf{T^{-1}} = \frac{1}{16}\begin{bmatrix} 7 & -1 \\ -1 & 7 \end{bmatrix}],$$

    (b) $\mathcal{L} = \mathcal{P}_2$,
    $f_1 = x^2 + 2x + 1$, $f_2 = x^2 + 2x - 1$, $f_3 = x^2 - 2x - 1$,
    $g_1 = x^2$, $g_2 = x$, $g_3 = 1$,

    $$[\mathbf{T} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & -2 \\ 2 & -1 & 0 \end{bmatrix}, \mathbf{T^{-1}} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & -2 \\ 1 & -1 & -1 \end{bmatrix}],$$

    (c) $\mathcal{L} = \mathcal{M}_{3,2}$,

    $$\mathbf{F_1} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{F_2} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{F_3} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}, \mathbf{F_4} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}, \mathbf{F_5} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \end{bmatrix},$$

    $$\mathbf{F_6} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

    $$\mathbf{G_1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{G_2} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{G_3} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{G_4} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{G_5} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix},$$

    $$\mathbf{G_6} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

    $$[\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 3 & -1 & 1 & 0 & 0 & 0 \\ -5 & 3 & -1 & 1 & 0 & 0 \\ 11 & -5 & 3 & -1 & 1 & 0 \\ -21 & 11 & -5 & 3 & -1 & 1 \end{bmatrix}, \mathbf{T^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{bmatrix}].$$

16. Určete matici $\mathbf{A}$ lineárního operátoru $L: \mathcal{L} \longrightarrow \mathcal{L}$ v bázi $f_1, f_2, \dots$ prostoru $\mathcal{L}$, matici $\mathbf{B}$ téhož lineárního operátoru L v bázi $g_1, g_2, \dots$ prostoru $\mathcal{L}$ a $\mathbf{T}$ matici přechodu od báze $f_1, f_2, \dots$ k bázi $g_1, g_2, \dots$. Určete $\mathbf{T^{-1}AT}$.

    (a) $\mathcal{L} = \mathbb{R}_2$,
    $L([a,b]^T) = [2a - b, a + 3b]^T$,
    $f_1 = [1,4]^T$, $f_2 = [4,1]^T$,

$g_1 = [2,0]^T,\ g_2 = [2,1]^T,$

$$[\mathbf{A} = \tfrac{1}{5}\begin{bmatrix} 18 & 7 \\ -7 & 7 \end{bmatrix}, \mathbf{B} = \tfrac{1}{2}\begin{bmatrix} 0 & -7 \\ 4 & 10 \end{bmatrix}, \mathbf{T} = \tfrac{1}{15}\begin{bmatrix} -2 & 2 \\ 8 & 7 \end{bmatrix}, \mathbf{T^{-1}AT} = \mathbf{B},]$$

(b) $\mathcal{L} = \mathcal{P}_2,$
$L(ax^2+bx+c) = (a-b+2c)x^2 + (3a+b-c)x + (4a+c),$
$f_1 = x^2+2x-1,\ f_2 = x+2,\ f_3 = 1,$
$g_1 = x^2+x,\ g_2 = x+1,\ g_3 = x^2+1,$

$$[\mathbf{A} = \begin{bmatrix} -3 & 3 & 2 \\ 12 & -7 & -5 \\ -24 & 19 & 13 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \\ 3 & -1 & 6 \end{bmatrix},$$
$$\mathbf{T^{-1}AT} = \mathbf{B}].$$