Domácí cvičení č. 1

1. Napište reálný rozklad a rozklad na kořenové činitele polynomu $p(x)$.

   (a) $p(x) = x^3 + 2x^2 - 5x - 6$,
   $[p(x) = (x+3)(x-2)(x+1)]$,

   (b) $p(x) = x^5 - 4x^4 + 8x^3 - 14x^2 + 15x - 6$,
   $[p(x) = (x-2)(x-1)^2(x^2+3) = (x-2)(x-1)^2(x-\sqrt{3}i)(x+\sqrt{3}i)]$,

   (c) $p(x) = x^4 - 1$,
   $[(x-1)(x+1)(x^2+1) = (x-1)(x+1)(x-i)(x+i)]$,

   (d) $p(x) = x^6 + 64$,
   $[(x^2+4)(x^2-2\sqrt{3}x+4)(x^2+2\sqrt{3}x+4) =$
   $(x-2i)(x+2i)(x-\sqrt{3}-i)(x-\sqrt{3}+i)(x+\sqrt{3}-i)(x+\sqrt{3}+i)]$,

   (e) $p(x) = x^4 + 5x^2 + 6$,
   $[(x^2+3)(x^2+2) = (x-\sqrt{3}i)(x+\sqrt{3}i)(x-\sqrt{2}i)(x+\sqrt{2}i)]$.

2. Jsou dány matice

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 5 & 1 & 2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 1 & 4 & -5 \\ -2 & 1 & 3 & 2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 2 & 5 \\ 1 & 2 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}.$$

   Určete matice

   (a) $\mathbf{A} + 2\mathbf{B}$, $-3\mathbf{A} + 4\mathbf{B}$, $\mathbf{A} + \mathbf{D}$, $\mathbf{D} - \mathbf{C}$.
   Řešení: $\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 1 & 4 & 5 & -6 \\ 1 & 3 & 8 & 4 \end{bmatrix}$, $-3\mathbf{A} + 4\mathbf{B} = \begin{bmatrix} -3 & -2 & 25 & -32 \\ -23 & 1 & 6 & 8 \end{bmatrix}$,
   $\mathbf{A} + \mathbf{D}$ - nelze, $\mathbf{D} - \mathbf{C}$ - nelze.

   (b) $\mathbf{AB}$, $\mathbf{AC}$, $\mathbf{CA}$, $\mathbf{BC}$, $\mathbf{CB}$, $\mathbf{AD}$, $\mathbf{DA}$, $\mathbf{BD}$, $\mathbf{DB}$.
   Řešení: $\mathbf{AB}$ - nelze, $\mathbf{AC} = \begin{bmatrix} -1 & 3 \\ 18 & 24 \end{bmatrix}$, $\mathbf{CA} = \begin{bmatrix} 13 & 8 & -5 & 12 \\ 19 & 2 & 11 & -4 \\ 27 & 9 & 4 & 8 \\ 11 & 4 & 1 & 4 \end{bmatrix}$,
   $\mathbf{BC} = \begin{bmatrix} 2 & 14 \\ 1 & 19 \end{bmatrix}$, $\mathbf{CB} = \begin{bmatrix} -4 & 5 & 18 & -11 \\ -8 & 3 & 8 & 13 \\ -10 & 7 & 23 & 0 \\ -4 & 3 & 10 & -1 \end{bmatrix}$, $\mathbf{AD}$ - nelze,
   $\mathbf{DA} = \begin{bmatrix} -3 & 3 & -8 & 8 \\ 28 & 11 & 1 & 12 \end{bmatrix}$, $\mathbf{BD}$ - nelze, $\mathbf{DB} = \begin{bmatrix} 2 & 1 & 5 & -12 \\ -10 & 8 & 27 & -5 \end{bmatrix}$.

3. Určete determinant matice $\mathbf{A}$.

   (a) $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $[\det \mathbf{A} = 22]$,

(b) $\mathbf{A} = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 2 \\ -3 & 2 & -1 \end{bmatrix}$, $[\det \mathbf{A} = -89]$,

(c) $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -1 & 3 \\ -1 & 1 & 4 \end{bmatrix}$, $[\det \mathbf{A} = 0]$,

(d) $\mathbf{A} = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ a & b & c & d \\ 1 & 1 & 1 & 3 \end{bmatrix}$, $[\det \mathbf{A} = -4a - 4b + 20c - 4d]$,

(e) $\mathbf{A} = \begin{bmatrix} 2 & 1 & -3 & 4 \\ -3 & -2 & 2 & -1 \\ 4 & 3 & -2 & 6 \\ -1 & -2 & 2 & -3 \end{bmatrix}$, $[\det \mathbf{A} = 48]$,

(f) $\mathbf{A} = \begin{bmatrix} 2 & 3 & -3 & 2 & 4 \\ -3 & -2 & 4 & -3 & -2 \\ 4 & 5 & -4 & 3 & 5 \\ 3 & 4 & -2 & 4 & 3 \\ -2 & -4 & 2 & -3 & -5 \end{bmatrix}$, $[\det \mathbf{A} = -12]$.

(g) $\mathbf{A} = \begin{bmatrix} 2 & 3 & -2 & 3 & 4 \\ -3 & 2 & -4 & 4 & 5 \\ -3 & -4 & 2 & -2 & -4 \\ 2 & 3 & -3 & 4 & 3 \\ 4 & 2 & -5 & 3 & 5 \end{bmatrix}$, $[\det \mathbf{A} = 228]$.