Domácí cvičení č.7

21. Určete vlastní čísla, vlastní vektory a Jordanův kanonicky tvar matice $\mathbf{A}$.

(a) $\mathbf{A} = \begin{bmatrix} 14 & 4 \\ -30 & -8 \end{bmatrix}$,

$[\lambda_1 = 2,\ \lambda_2 = 4,\ h_1 = [-1,3]^T,\ h_2 = [-2,5]^T,\ \mathbf{J} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}]$,

(b) $\mathbf{A} = \begin{bmatrix} 13 & 5 \\ -34 & -13 \end{bmatrix}$,

$[\lambda_1 = i,\ \lambda_2 = -i,\ h_1 = [-13-i,34]^T,\ h_2 = [-13+i,34]^T,\ \mathbf{J} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}]$,

(c) $\mathbf{A} = \begin{bmatrix} 10 & 5 & -3 \\ 17 & 10 & -9 \\ 39 & 21 & -16 \end{bmatrix}$,

$[\lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 3,\ h_1 = [-1,4,3]^T,\ h_2 = [1,-1,1]^T,\ h_3 = [2,-1,3]^T,$

$\mathbf{J} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}]$,

(d) $\mathbf{A} = \begin{bmatrix} 12 & -18 & -6 \\ -28 & 42 & 14 \\ 104 & -156 & -52 \end{bmatrix}$,

$[\lambda_{1,2} = 0,\ \lambda_3 = 2,\ h_1 = [3,2,0]^T,\ h_2 = [1,0,2]^T,\ h_3 = [-3,7,-26]^T,$

$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}]$,

(e) $\mathbf{A} = \begin{bmatrix} 23 & 8 & -7 \\ -76 & -26 & 24 \\ -13 & -4 & 5 \end{bmatrix}$,

$[\lambda_{1,2} = 0,\ \lambda_3 = 2,\ h_1 = [1,-2,1]^T,\ h_2 = [-1,7,5]^T,\ \mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}]$,

(f) $\mathbf{A} = \begin{bmatrix} -59 & 75 & 27 \\ 41 & -53 & -19 \\ -241 & 309 & 111 \end{bmatrix}$,

$[\lambda_1 = 0,\ \lambda_2 = 1,\ \lambda_3 = -2,\ h_1 = [-3,7,-26]^T,\ h_2 = [1,-1,5]^T,\ h_3 = [2,-1,7]^T$

$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}]$.

22. K matici $\mathbf{A}$ určete Jordanův kanonický tvar $\mathbf{J}$ a matici $\mathbf{T}$. Ověřte, že platí $\mathbf{A} = \mathbf{TJT^{-1}}$.

(a) $\mathbf{A} = \begin{bmatrix} 14 & 4 \\ -30 & -8 \end{bmatrix}$,

$[\mathbf{J} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} -1 & -2 \\ 3 & 5 \end{bmatrix}]$,

(b) $\mathbf{A} = \begin{bmatrix} 13 & 5 \\ -34 & -13 \end{bmatrix}$,

$[\mathbf{J} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \mathbf{T} = \begin{bmatrix} -13-i & -13+i \\ 34 & 34 \end{bmatrix}]$,

(c) $\mathbf{A} = \begin{bmatrix} 10 & 5 & -3 \\ 17 & 10 & -9 \\ 39 & 21 & -16 \end{bmatrix}$,

$[\mathbf{J} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} -1 & 1 & 2 \\ 4 & -1 & -1 \\ 3 & 1 & 3 \end{bmatrix}]$,

(d) $\mathbf{A} = \begin{bmatrix} 12 & -18 & -6 \\ -28 & 42 & 14 \\ 104 & -156 & -52 \end{bmatrix}$,

$[\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} 3 & 1 & -3 \\ 2 & 0 & 7 \\ 0 & 2 & -26 \end{bmatrix}]$,

(e) $\mathbf{A} = \begin{bmatrix} -59 & 75 & 27 \\ 41 & -53 & -19 \\ -241 & 309 & 111 \end{bmatrix}$,

$[\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} -3 & 1 & 2 \\ 7 & -1 & -1 \\ -26 & 5 & 7 \end{bmatrix}]$.