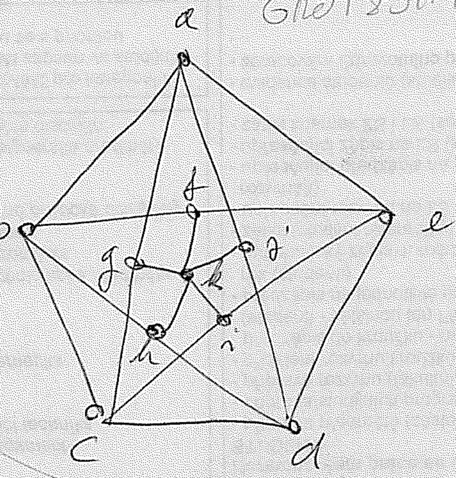
MYCIELSKI
GRÖTZSCH

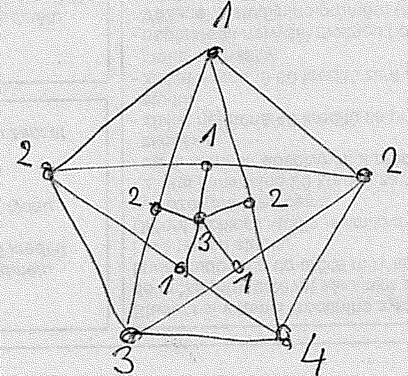
Urči $\chi(G)$:



1) Má lichou kružnici $\Rightarrow \chi(G) \geq 3$

2) $\Delta(G) = 5 \Rightarrow$ (BROOKS) $\chi(G) \leq 5$

3) Snadno najdu 4-obarvení (např. sekvenčním barvením nejmenší barvou z „shora dolů") $\rightarrow$



tedy $\chi(G) = 3$ nebo 4.

4) Je G 3-obarvitelný – chci dokázat, že ne. Nechť Ano:

– horní uzel (a) musí mít barvu 1,

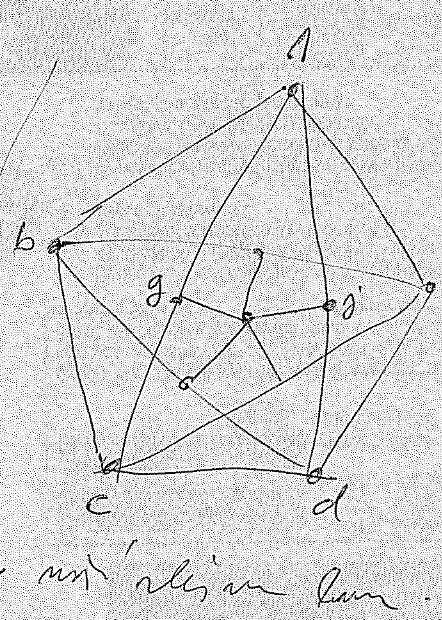
– Nemůže být současně $\chi(b) \neq \chi(g)$ i $\chi(g') \neq \chi(e)$.

Když ano: pak

c soused b i g $\Rightarrow \chi(c) = 1$

d soused g' i e $\Rightarrow \chi(d) = 1$

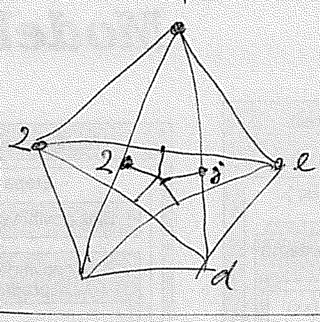
ale to nejde.



Tedy: b a g nebo g' a e mají stejnou barvu.

Symetrie $\Rightarrow$ lze předp. $\chi(b) = \chi(g) = 2$.

Takže víme: $\longrightarrow$



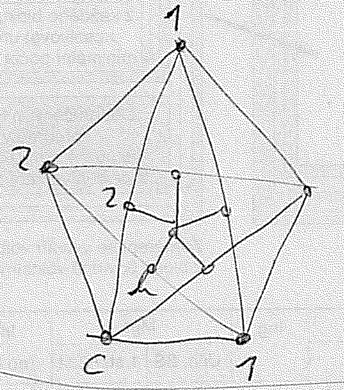
– ukážu, že $\chi(j) = \chi(e)$.

Kdyby $\chi(j) \neq \chi(e)$,

pak by $\chi(d) = 1$

$\Rightarrow \chi(h) = 3$

$\Downarrow \chi(c) = 3$



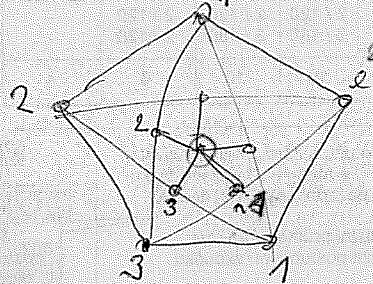
Ale pak

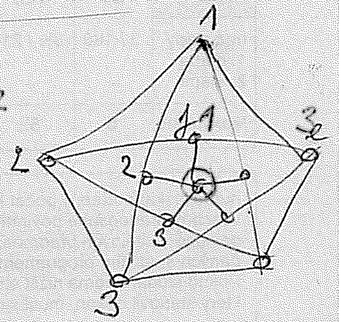
$\chi(e) = 2 \Rightarrow \chi(i) = 1$

$\chi(e) = 3 \Rightarrow \chi(f) = 1$

a v obou případech

nelze obarvit f/k.

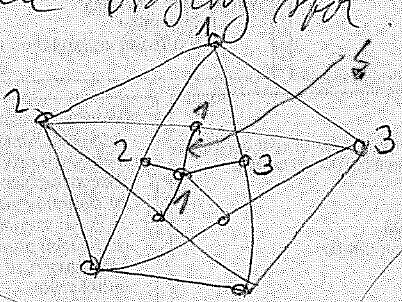




Tedy $\chi(j) = \chi(e)$.

Ale pak mi taky vyjde vždycky spor.

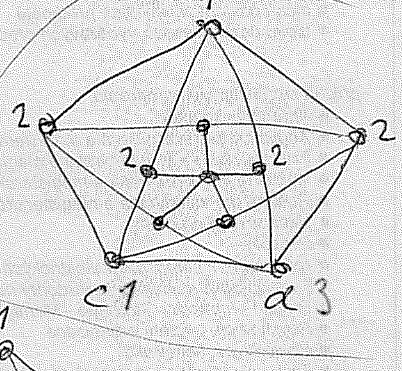
– $\chi(j) = \chi(e) = 3$:



– $\chi(j) = \chi(e) = 2$:

je $\chi(c) \neq \chi(d)$;

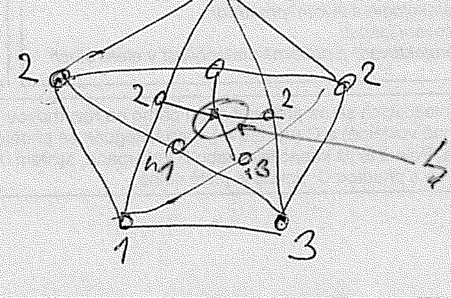
symetrie $\Rightarrow$ lze předp. $\chi(c) = 1$, $\chi(d) = 3$



ale pak

$\chi(h) = 1$, $\chi(i) = 3$

a mám spor na $\chi(k)$.



Tedy $\chi(G) = 4$.