c) $a_n = \frac{(-1)^n}{\ln^2(n+1)} (1 - \cos\frac{1}{\sqrt{n}})$ / k. abs. /

d) $a_n = \frac{(-1)^{n+1}}{2n-1}$ / k. rel. / e) $a_n = \frac{\sin n\alpha}{n^2}$ / k. abs. /

f) $a_n = \frac{(-1)^{n+1} n^3}{2^n}$ / k. abs. / g) $a_n = \frac{\cos(n\pi/4)}{(n+2)\ \ln^3(n+3)}$ /k.a. /

h) $a_n = \frac{\mathrm{arctg}(-n)^n}{\sqrt[4]{2n^6+3n+1}}$ / k. a. / i) $a_n = \frac{(-1)^n}{n - \ln n}$ / k. r. /

j) $a_n = \frac{(-1)^n\, n}{(n+2)\sqrt[4]{n+1}}$ / k. r. / k) $a_n = (-1)^{n+1}\frac{1}{n\sqrt{n^2+1}}$ / d /

l) $a_n = (-1)^{n-1}\frac{\sqrt[3]{n+1}}{\sqrt{n}+2}$ / k. r. / m) $a_n = (-1)^n\frac{n^2\, 2^n}{3^n+1}$ / k. a. /

6. Buď $\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} a_n \cdot \sum_{n=1}^{\infty} b_n$. Najděte c_n, je-li

a) $a_n = a^{n-1}$, $b_n = b^{n-1}$ $|a| < 1$, $|b| < 1$ / $c_n = \frac{b^n - a^n}{b - a}$ pro $b \neq a$

$= n\, a^{n-1}$ pro $a = b$

b) $a_n = q^{n-1}$, $b_n = \frac{q^n}{n(n+1)}$ $|q| < 1$ / $c_n = \frac{n}{n+1} q^n$ /

c) $a_n = \frac{2^{n-1}}{(n-1)!}$, $b_n = \frac{3^{n-1}}{(N-1)!}$ / $c_n = \frac{5^{n-1}}{(n-1)!}$ /

Funkční řady.

Najděte obor konvergence řady $\sum_{n=1}^{\infty} u_n(x)$, je-li

a) $u_n(x) = \ln^n x$ / $\frac{1}{e} < x < e$ /

b) $u_n(x) = \frac{(-1)^n}{2n+1}\left(\frac{1-x}{1+x}\right)^n$ / $\langle 0, \infty)$ /

c) $u_n(x) = \frac{1}{1+x^n}$ / $x \in \mathbb{R} - \langle -1, 1\rangle$ /

d) $u_n(x) = \frac{x^n}{1+x^{2n}}$ / $x \in \mathbb{R} - \{-1, 1\}$ /

e) $u_n(x) = \frac{(-1)^{n+1}}{x^n}$ / $|x| > 1$ / f) $u_n(x) = e^{-nx}$ / $x > 0$ /

g) $u_n(x) = \frac{\cos nx}{e^{nx}}$ / $x > 0$ / h) $u_n(x) = (5 - x^2)^n$ / $2 < |x| < \sqrt{6}$ /

i) $u_n(x) = n^{-\ln x^2}$ / $|x| > \sqrt{e}$ / j) $u_n(x) = n^2 e^{-nx^2}$ / $x \in \mathbb{R} - \{0\}$ /

k) $u_n(x) = \frac{x^n}{1 - x^n}$ / $|x| < 1$ /

- Rozhodněte o stejnoměrné konvergenci posloupnosti $\{ f_n(x) \}$, je-li

a) $f_n(x) = \frac{n^2}{n^2 + x^2}$ $x \in \mathbb{R}$ / nestej. / $x \in \langle -1, 1 \rangle$ / stej. /

b) $f_n(x) = x^n$ $x \in \langle 0, 1 \rangle$ / nestej. / $x \in \langle 0, \frac{1}{2} \rangle$ / stej. /

c) $f_n(x) = \frac{\operatorname{arctg} nx}{\sqrt{n + x}}$ $x \in \langle 0, +\infty)$ / stej. /

d) $f_n(x) = x^n - x^{n+1}$ $x \in \langle 0, 1 \rangle$ / stej. /

e) $f_n(x) = x^n - x^{2n}$ $x \in \langle 0, 1 \rangle$ / nestej. /

f) $f_n(x) = \frac{nx}{1 + n^2x^2}$ $x \in \langle 0, 2 \rangle$ / nestej. /

g) $f_n(x) = \frac{2x}{1 + n^2x^2}$ $x \in \mathbb{R}$ / stej. /

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√ h) $f_n(x) = \sqrt{x + \frac{1}{n}} - \sqrt{x}$ $x \in \langle 0, +\infty)$ / stej. / lim → 0 → 0 ⇒ STEJN. KONV.

i) $f_n(x) = e^{n(x - 1)}$ $x \in (0, 1)$ / nestej. /

j) $f_n(x) = \operatorname{arctg} nx$ $x \in (0, +\infty)$ / nestej. /

k) $f_n(x) = x \operatorname{arctg} nx$ $x \in (0, +\infty)$ / stej. /

- Dokažte stejnoměrnou konvergenci $\sum_{n=1}^{\infty} u_n(x)$, je-li

a) $u_n(x) = \frac{1}{x^2 + n^2}$, $x \in \mathbb{R}$ b) $u_n(x) = \frac{(-1)^n}{x + 2^n}$, $x \geq 0$

c) $u_n(x) = \frac{x}{1 + n^4x^2}$, $x \in \mathbb{R}$ d) $u_n(x) = \frac{\sin nx}{\sqrt[3]{n^4 + x^4}}$, $x \in \mathbb{R}$

e) $u_n(x) = \frac{nx}{1 + n^5x^2}$, $x \in \mathbb{R}$ f) $u_n(x) = \operatorname{arctg} \frac{2x}{x^2 + n^3}$, $x \in \mathbb{R}$

g) $u_n(x) = \frac{\cos nx}{n^2}$, $x \in \mathbb{R}$ h) $u_n(x) = \frac{x^2 \sin(n\sqrt{x})}{1 + n^3x^4}$, $x \geq 0$

i) $u_n(x) = \left(\operatorname{arctg} \frac{x}{x^2 + n^2} \right)^2$, $x \geq 0$

j) $u_n(x) = \ln\left(1 + \frac{x^2}{n \ln^2 n}\right)$ $n \geq 2$, $|x| \leq a$, $(a > 0)$

k) $u_n(x) = \frac{\sin(x^2/n)}{x^2\sqrt{n} + 1}$, $|x| \leq a$ $(a > 0)$

l) $u_n(x) = \frac{\sin(x/n)\sin 2nx}{x^2 + 4n}$, $x \in \mathbb{R}$

m) $u_n(x) = \frac{n^2}{\sqrt{n!}}(x^n + x^{-n})$, $\frac{1}{2} \leq |x| \leq 2$

n) $u_n(x) = x^2 e^{-nx}$ $\varepsilon \leq x \leq a$ $(\varepsilon, a > 0, \varepsilon < a)$

Mocninné řady.

1. Najděte poloměr konvergence řady $\sum_{n=0}^{\infty} a_n x^n$, je-li

a) $a_n = \frac{1}{n^2}$ / 1 / b) $a_n = \frac{1}{n!}$ / ∞ / c) $a_n = \frac{(1+i)^n}{n\,2^n}$ / $\sqrt{2}$ /

d) $a_n = \alpha^{n^2}$ $(0 < \alpha < 1)$ / ∞ /

e) $a_n = \frac{a^n}{n} + \frac{b^n}{n^2}$ $(a, b > 0)$ / $\min\left(\frac{1}{a}, \frac{1}{b}\right)$ /

f) $a_n = \frac{3^{-\sqrt{n}}}{\sqrt{n^2 + 1}}$ / 1 / g) $a_n = \frac{1}{a^n + b^n}$ $(a, b > 0)$ / $\max(a, b)$ /

h) $a_n = (-1)^{n-1}\left\{\frac{2^n (n!)^2}{(2n+1)!}\right\}^p$ / 2^p /

i) $a_n = \frac{(-1)^{n-1}}{n!}\left(\frac{n}{e}\right)^n$ / 1 /

j) $a_n = \frac{a(a+1)\cdot\ldots\cdot(a+n-1)\,b(b+1)\cdot\ldots\cdot(b+n-1)}{n!\,c(c+1)\cdot\ldots\cdot(c+n-1)}$ / 1 /

2. Najděte poloměr konvergence řady

a) $\sum_{n=1}^{\infty} \frac{[3 + (-1)^n]^n}{n} x^n$ / $\frac{1}{4}$ / b) $\sum_{n=0}^{\infty} 5^n x^{3n}$ / $\frac{1}{\sqrt[3]{5}}$ /

c) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n} x^{2n}$ / $\sqrt{\frac{e}{2}}$ / d) $\sum_{n=0}^{\infty} 3^n (n^3 + 2) x^{2n}$ / $\frac{1}{\sqrt{3}}$ /

3. Najděte obor konvergence mocninné řady $\sum_{n=0}^{\infty} a_n (x - x_0)^n$, je-li

a) $a_n = \frac{1}{n\sqrt{n}}$, $x_o = 1$ / $\langle 0, 2\rangle$ / b) $a_n = \left(\frac{2n-1}{3n+2}\right)^n$, $x_o = -2$ / $\left(-\frac{7}{2}, \frac{-1}{2}\right)$ /

c) $a_n = \frac{(-1)^n}{2n+1}$, $x_o = 0$ / $(-1, 1\rangle$ /

d) $a_n = \frac{1}{\sqrt[3]{n}\,3^n}$, $x_o = 1$ / $\langle -2, 4)$ /

e) $a_n = \sqrt{\frac{n^4+3}{n^3+4n}}$, $x_o = -2$ / $(-3, -1)$ /

podobnej jsme dělali → f) $a_n = \frac{5^n + (-3)^n}{n+1}$, $x_o = 0$ / $\left\langle -\frac{1}{5}, \frac{1}{5}\right)$ /

g) $a_n = \frac{1}{\sqrt{n+1}} \ln \frac{3n-2}{3n+2}$, $x_o = -1$ / $\langle -2, 0\rangle$ /

h) $a_n = \frac{\sqrt[3]{2n+1} - \sqrt[3]{2n-1}}{\sqrt{n}}$, $x_o = -3$ / $\langle -4, -2\rangle$ /

i) $a_n = \sqrt[n]{a} - 1$, $x_o = 0$, $a > 0$, $a \neq 1$ / $\langle -1, 1)$ /

j) $a_n = \frac{3^{-\sqrt{n}}}{\sqrt{n^2+n+1}}$, $x_o = 1$ / $\langle 0, 2\rangle$ /

4. Najděte rozvoj funkce $f(x)$ v mocninnou řadu

a) $f(x) = e^{-x^2}$ / $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$; $x \in \mathbb{R}$ /

b) $f(x) = \cos^2 x$ / $1 + \sum_{n=1}^{\infty} (-1)^n \frac{2^{2n-1}}{(2n)!} x^{2n}$; $x \in \mathbb{R}$ /

c) $f(x) = \sin 3x \sin 5x$ / $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n)!} (1 - 2^{4n}) x^{2n}$; $x \in \mathbb{R}$ /

d) $f(x) = \sin^3 x$ / $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3(3^{2n}-1)}{4(2n+1)!} x^{2n+1}$; $x \in \mathbb{R}$ /

e) $f(x) = \frac{x^2}{(1+x)^2}$ / $\sum_{n=0}^{\infty} (-1)^n (n+1) x^{n+2}$; $x \in (-1, 1)$ /

f) $f(x) = \frac{5x-4}{x+2}$ / $-2 + \sum_{n=1}^{\infty} \frac{7(-1)^{n-1}}{2^n} x^n$; $x \in (-2, 2)$ /

g) $f(x) = \frac{1}{x^2 - 2x - 3}$ / $-\frac{1}{4}\sum_{n=0}^{\infty} \frac{1 + (-1)^n 3^{n+1}}{3^{n+1}} x^n$; $x \in (-1, 1)$ /

h) $f(x) = \ln\sqrt{\frac{1 + x}{1 - x}}$ / $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n + 1}$; $x \in (-1, 1)$ /

i) $f(x) = \ln \frac{3 - 2x}{2 + 3x}$ / $\ln \frac{3}{2} + \sum_{n=1}^{\infty} \left\{ (-\frac{3}{2})^n - (\frac{2}{3})^n \right\} \frac{x^n}{n}$; $x \in (-\frac{2}{3}, \frac{2}{3}\rangle$ /

j) $f(x) = \frac{1}{\sqrt{1 - x^2}}$ / $1 + \sum_{n=1}^{\infty} \frac{(2n - 1)!!}{(2n)!!} x^{2n}$; $x \in (-1, 1)$ /

k) $f(x) = \sqrt{1 + x^2}$ / $1 + \frac{x^2}{2} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} (2n - 3)!!}{(2n)!!} x^{2n}$; $x \in \langle -1, 1\rangle$ /

l) $f(x) = (1 - x^2)^{-\frac{3}{2}}$ / $\sum_{n=0}^{\infty} \frac{(2n + 1)!!}{(2n)!!} x^{2n}$; $x \in (-1, 1)$ /

m) $f(x) = \frac{x}{\sqrt{1 - 2x}}$ / $x + \sum_{n=1}^{\infty} \frac{(2n - 1)!!}{n!} x^{n+1}$; $x \in (-\frac{1}{2}, \frac{1}{2})$ /

n) $f(x) = (1 + x^2) \operatorname{arctg} x$ / $x + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2 - 1} x^{2n+1}$; $x \in \langle -1, 1\rangle$

5. Najděte rozvoj funkce $f(x)$ v mocninnou řadu.

a) $f(x) = \ln(x + \sqrt{1 + x^2})$ / $x + \sum_{n=1}^{\infty} \frac{(-1)^n (2n - 1)!!}{(2n)!!} \frac{x^{2n+1}}{2n + 1}$; $x \in \langle -1, 1\rangle$ /

b) $f(x) = \arcsin x$ / $x + \sum_{n=1}^{\infty} \frac{(2n - 1)!!}{(2n)!!} \frac{x^{2n+1}}{2n + 1}$; $x \in \langle -1, 1\rangle$ /

c) $f(x) = \operatorname{arctg} \frac{x + 3}{x - 3}$ / $-\frac{\pi}{4} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^{2n+1}} \frac{x^{2n+1}}{2n + 1}$; $x \in \langle -3, 3\rangle$ /

d) $f(x) = \frac{1}{1 - x - x^2}$ / $\sum_{n=0}^{\infty} a_n x^n$; $a_n = \frac{1}{\sqrt{5}}\left\{ (\frac{\sqrt{5} + 1}{2})^{n+1} + (-1)^n (\frac{\sqrt{5} - 1}{2})^{n+1} \right\}$, $|x| < \frac{\sqrt{5} - 1}{2}$

e) $f(x) = \frac{1}{1 + x + x^2}$ / $\frac{2}{\sqrt{3}} \sum_{n=0}^{\infty} x^n \sin \frac{2\pi(n + 1)}{3}$; $x \in (-1, 1)$ /

f) $f(x) = \dfrac{x\cos\alpha - x^2}{1 - 2x\cos\alpha + x^2}$ / $\sum\limits_{n=1}^{\infty} x^n \cos n\alpha$; $x \in (-1, 1)$ /

g) $f(x) = \frac{1}{4} \ln \frac{1+x}{1-x} + \frac{1}{2} \operatorname{arctg} x$ / $\sum\limits_{n=0}^{\infty} \dfrac{x^{4n+1}}{4n+1}$; $x \in (-1, 1)$ /

h) $f(x) = x \operatorname{arctg} x - \ln\sqrt{1+x^2}$ / $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \dfrac{x^{2n}}{2n(2n-1)}$; $x \in \langle -1, 1 \rangle$ /

i) $f(x) = x \arcsin x + \sqrt{1-x^2}$ / $1 + \frac{x^2}{2} + \sum\limits_{n=1}^{\infty} \dfrac{(2n-1)!!}{(2n+2)!!} \dfrac{x^{2n+2}}{2n+1}$; $x \in \langle -1, 1 \rangle$ /

j) $f(x) = \dfrac{\ln(1+x)}{1+x}$ / $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right) x^n$; $x \in (-1, 1)$ /

k) $f(x) = \dfrac{e^x}{1-x}$ / $\sum\limits_{n=0}^{\infty} \sum\limits_{k=0}^{n} \frac{1}{k!} x^n$; $x \in (-1, 1)$ /

l) $f(x) = \operatorname{arctg}^2 x$ / $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \left(1 + \frac{1}{3} + \dots + \frac{1}{2n-1}\right) \dfrac{x^{2n}}{n}$; $x \in \langle -1, 1 \rangle$ /

m) $f(x) = e^x \sin x$ / $\sum\limits_{n=1}^{\infty} \dfrac{2^{n/2} \sin(n\pi/4)}{n!} x^n$; $x \in \mathbb{R}$ /

n) $f(x) = e^x \cos x$ / $\sum\limits_{n=0}^{\infty} \dfrac{2^{n/2} \cos(n\pi/4)}{n!} x^n$; $x \in \mathbb{R}$ /

o) $f(x) = \left(\dfrac{\arcsin x}{x}\right)^2$ / $\sum\limits_{n=0}^{\infty} \dfrac{2^{2n+1} (n!)^2}{(2n+2)!} x^{2n}$; $|x| \leqq 1$ /

. Vypočtěte integrály

a) $\int\limits_0^x e^{-t^2}\, dt$ / $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{n!\,(2n+1)} x^{2n+1}$; $x \in \mathbb{R}$ /

b) $\int\limits_0^x \dfrac{\sin t}{t}\, dt$ / $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$; $x \in \mathbb{R}$ /

c) $\int\limits_0^x \dfrac{dt}{\sqrt{1-t^4}}$ / $x + \sum\limits_{n=1}^{\infty} \dfrac{(2n-1)!!\, x^{4n+1}}{(2n)!!\,(4n+1)}$; $x \in (-1, 1)$ /

d) $\int_0^x \frac{t^2\,dt}{\sqrt{1+t^2}}$ / $\frac{x^3}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+3}}{(2n+3)}$; $x \in \langle -1, 1 \rangle$ /

7. Sečtěte řady

a) $\sum_{n=0}^{\infty} \frac{\ln^n x}{n!}$ / x ; $x > 0$ / b) $\sum_{n=0}^{\infty} \frac{(-1)^n \ln^n x}{2^n\, n!}$ / $\frac{1}{\sqrt{x}}$; $x > 0$ /

~~c)~~ $\sum_{n=1}^{\infty} \frac{x^n}{n}$ / $\ln \frac{1}{1-x}$; $x \in \langle -1, 1)$ /

~~d)~~ $\sum_{n=1}^{\infty} n\, x^{n+1}$ / $\frac{x^2}{(1-x)^2}$; $x \in (-1, 1)$ /

e) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{2n+1}$ / $\frac{\operatorname{arctg} x}{x}$; $x \in \langle -1, 1 \rangle$ /

f) $\sum_{n=1}^{\infty} n(n+2)\, x^n$ / $\frac{x(3-x)}{(1-x)^3}$; $x \in (-1, 1)$ /

g) $\sum_{n=0}^{\infty} \frac{(3n+1)\, x^{3n}}{n!}$ / $(1 + 3x^2)\, e^{x^3}$; $x \in \mathbb{R}$ /

h) $\sum_{n=0}^{\infty} \frac{(-1)^n (2n^2+1)}{(2n)!} x^{2n}$ / $(1 - \frac{x^2}{2}) \cos x - \frac{x}{2} \sin x$; $x \in \mathbb{R}$ /

i) $\sum_{n=0}^{\infty} \frac{n^2+1}{2^n\, n!} x^n$ / $e^{x/2} (\frac{x^2}{4} + \frac{x}{2} + 1)$; $x \in \mathbb{R}$ /

Fourierovy řady.

1. Najděte Fourierovu řadu funkcí $f_n(x) = \sin^n x$ a $g_n(x) = \cos^n x$ pro $n = 2, 3, 4, 5$.

/ $f_2(x) = \frac{1}{2} - \frac{1}{2} \cos 2x$; $g_2(x) = \frac{1}{2} + \frac{1}{2} \cos 2x$; $f_3(x) = \frac{3}{4} \sin x - \frac{1}{4} \sin 3x$; $g_3(x) = \frac{3}{4} \cos x + \frac{1}{4} \cos 3x$; $f_4(x) = \frac{3}{4} - \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$; $g_4(x) = \frac{3}{4} + \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$; $f_5(x) = - \frac{5}{8} \sin x + \frac{5}{16} \sin 3x - \frac{1}{16} \sin 5x$; $g_5(x) = \frac{5}{8} \cos x + \frac{5}{16} \cos 3x + \frac{1}{16} \cos 5x$ /

• Najděte Fourierovu řadu funkce $f(x)$ na intervalu $(-\pi, \pi)$, je-li

a) $f(x) = x$ $\quad / \; 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n} \; /$

b) $f(x) = 1$ pro $0 \leq x \leq \pi$
$\quad\;\; = 0$ pro $-\pi \leq x \leq 0$ $\quad / \; \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$

c) $f(x) = |x|$. Výsledku využijte k sečtení řady $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$

$/ \; \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2} \; ; \; \frac{\pi^2}{8} \; /$

d) $f(x) = \pi^2 - x^2$. Výsledku využijte k sečtení řad $\sum_{n=1}^{\infty} \frac{1}{n^2}$ / $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$/ \; \frac{2}{3}\pi^2 + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx \; ; \; \frac{\pi^2}{6} , \frac{\pi^2}{12} \; /$

e) $f(x) = \operatorname{sign} x$. Výsledku využijte k sečtení řady $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

$/ \; \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} \; ; \; \frac{\pi}{4} \; /$

f) $f(x) = \sin ax \quad a \notin \mathbb{Z}$ $\quad / \; \frac{2 \sin \pi a}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n \sin nx}{n^2 - a^2} \; /$

g) $f(x) = \cos ax \quad a \notin \mathbb{Z}$ $\quad / \; \frac{2 \sin \pi a}{\pi} \left\{ \frac{1}{2a} + \sum_{n=1}^{\infty} (-1)^n \frac{a \cos nx}{a^2 - n^2} \right\} /$

h) $f(x) = e^{ax} \quad a \neq 0$ $\quad / \; \frac{2}{\pi} \operatorname{sh} a\pi \left\{ \frac{1}{2a} + \sum_{n=1}^{\infty} \frac{(-1)^n}{a^2 + n^2} (a \cos nx - n \sin nx) \right\} /$

i) $f(x) = \frac{q \sin x}{1 - 2q \cos x + q^2}$ $\quad |q| < 1$ $\quad / \; \sum_{n=1}^{\infty} q^n \sin nx$;
zaveďte $e^{ix} = z$ /

• Najděte Fourierovu řadu funkce $f(x)$, je-li

a) $f(x) = \frac{\pi - x}{2}$, $x \; (0, 2\pi)$ $\quad / \; \sum_{n=1}^{\infty} \frac{\sin nx}{n} \; /$

b) $f(x) = x$, $x \in (a, a+2l)$ $\quad / \; a+l + \frac{2l}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\sin \frac{n\pi a}{l} \cos \frac{n\pi x}{l} - \right.$
$\left. - \cos \frac{n\pi a}{l} \sin \frac{n\pi x}{l} \right) \; /$

c) $f(x) = x^2$, $x \in (0, 2\pi)$ / $\frac{4\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} - 4\pi \sum_{n=1}^{\infty} \frac{\sin nx}{n}$ /

d) $f(x) = e^{ax}$, $x \in (-h, h)$

/ $2 \operatorname{sh} ah \left\{ \frac{1}{2ah} + \sum_{n=1}^{\infty} (-1)^n \frac{ah \cos(n\pi x/h) - n \sin(n\pi x/h)}{(ah)^2 + (\pi n)^2} \right\}$ /

e) $f(x) = x \cos x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ / $\frac{16}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{(4n^2 - 1)^2} \sin 2nx$ /

f) $f(x) = e^x - 1$, $x \in (0, 2\pi)$

/ $\frac{e^{2\pi} - 1}{\pi} \left\{ \frac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{\cos nx}{1+n^2} - \frac{n \sin nx}{1+n^2} \right) \right\} - 1$ /

4. Najděte Fourierovu řadu funkce $f(x)$, je-li

a) $f(x) = \frac{\pi}{4} - \frac{x}{2}$, $x \in (0, \pi)$ (kosinová řada) / $\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2}$ /

b) $f(x) = x^2$, $x \in (0, \pi)$ (sinová řada)

/ $\frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \left\{ \frac{\pi^2}{n} + \frac{2}{n^2}\left[(-1)^n - 1 \right] \right\} \sin nx$ /

c) $f(x) = \sin ax$, $a \in \mathbb{Z}$, $x \in (0, \pi)$ (kosinová řada)

/ $\frac{4a}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{a^2 - (2n+1)^2}$ pro a sudé ;

$\frac{4a}{\pi} \left\{ \frac{1}{2a^2} + \sum_{n=1}^{\infty} \frac{\cos 2nx}{a^2 - 4n^2} \right\}$ pro a liché /

d) $f(x) = \cos ax$, $a \in \mathbb{Z}$, $x \in (0, \pi)$ (sinová řada)

/ $-\frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{a^2 - (2n+1)^2}$ pro a sudé ;

$-\frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n \sin 2nx}{a^2 - 4n^2}$ pro a liché /

e) $f(x) = x\left(\frac{\pi}{2} - x\right)$, $x \in \left(0, \frac{\pi}{2}\right)$ podle soustavy $\alpha) \left\{ \cos(2n-1)x \right\}$, $n \in \mathbb{N}$; $\beta) \left\{ \sin(2n-1)x \right\}$, $n \in \mathbb{N}$

/ α) $-2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \left\{ 1 + \frac{4(-1)^n}{(2n-1)\pi} \right\} \cos(2n-1)x$ /

β) $\sum_{n=1}^{\infty} \left\{ \frac{2(-1)^n}{(2n-1)^2} + \frac{8}{(2n-1)^3} \right\} \sin(2n-1)x$ /

. Integrací Fourierova rozvoje funkce $f(x) = x$ najděte rozvoj funkcí x^2, x^3, x^4, x^5 pro $x \in (-\pi, \pi)$

/ $x \sim 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$; $x^2 \sim \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} (-1)^n$;

$x^3 \sim 2 \sum_{n=1}^{\infty} (-1)^n \frac{6 - \pi^2 n^2}{n^3} \sin nx$;

$x^4 \sim \frac{\pi^4}{5} + 8 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{6 - \pi^2 n^2}{n^4} \cos nx$;

$x^5 \sim 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{120 - 20\pi^2 n^2 + \pi^4 n^4}{n^5} \sin nx$ /

Soustavy diferenciálních rovnic.

1. Pomocí metody postupných aproximací najděte několik prvých aproximací následujících rovnic.

a) $y' = y^2 + xy + x^2$, $y(0) = 1$ / $1 + x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + \frac{13}{24}x^4 + \frac{1}{4}x^5 + \frac{1}{18}x^6 + \frac{1}{63}x^7 + \dots$ /

b) $y' = y^3 - x$, $y(0) = 1$ / $1 + x + x^2 + \frac{1}{2}x^3 - \frac{1}{2}x^4 - \frac{3}{20}x^5 + \frac{1}{8}x^6 - \frac{1}{56}x^7 + \dots$ /

c) $y' = x^2y^2 - 1$, $y(0) = 1$ / $1 - x + \frac{x^3}{3} - \frac{1}{2}x^4 + \frac{1}{5}x^5 + \frac{1}{9}x^6 - \frac{2}{21}x^7 + \frac{1}{81}x^9 + \dots$ /

d) $y' = \frac{xy}{1 + x + y}$; $y(0) = 0$ / 0 /

e) $y' = \frac{1}{1-x} y$, $y(0) = 1$ / $1 - \ln(1-x) + \frac{1}{2}\ln^2(1-x) + \frac{1}{6}\ln^3(1-x) + \dots$; přesné řešení $y = \frac{1}{1-x}$ /